Correspondence

Measurement of the Conversion Conductances of Esaki Mixer Diodes*

In order to compare the theoretically predicted and the experimentally observed behavior of frequency converters using Esaki diodes, it is necessary to evaluate various mixer parameters, in particular, the conductance elements of the conversion matrix. As is well known, these conductance elements are certain Fourier coefficients of the periodic series representing the incremental time-varying diode conductance.[1] This conductance is produced by the large local oscillator (LO) voltage acting on the nonlinear diode. Although one can determine these element values using a numerical Fourier analysis, this approach is both time-consuming and laborious.[2] As an alternative, we suggest an experimental technique originally proposed by Dicke over ten years ago for use with conventional (positive-conductance) crystal mixer diodes.[1] Since this method is quite easy to apply in that it only involves several low-frequency bridge measurements and one standing-wave measurement, and since, apparently, it is not well known, we shall risk repetition by describing a simplified version of it which we have modified to apply to Esaki diodes.

To simplify the problem we assume that in the voltage and frequency range in which it is to be operated as a mixer, the Esaki diode can be represented adequately as a nonlinear conductance in parallel with a voltage independent junction capacitance. We also postulate that the LO voltage is sinusoidal and that all but three signal voltages generated in the mixing process can be considered terminated in short circuits. These three correspond to the input (RF) frequency ω_1, the output (IF) frequency ω_2, and the image frequency ω_3. Without loss of generality, we assume down conversion with the RF frequency above the LO frequency ω_0. That is, $\omega_2=\omega_1-\omega_0$, and $\omega_3=2\omega_0-\omega_1$. In this case the small-signal equivalent circuit of the mixer is that shown in Fig. 1.[1,2] The conductances g_0, g_1, g_2 are the first three Fourier coefficients of the

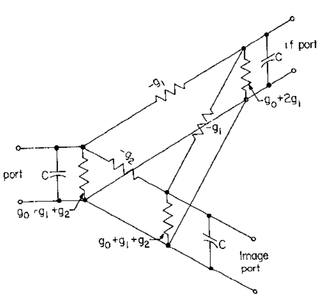

Fig. 1—Equivalent circuit of an Esaki mixer.

* Received by the PGMTT, July 17, 1961.

[1] H. Torrey and C. Whitmer, "Crystal Rectifiers," McGraw-Hill Book Co., Inc., New York, N. Y. 1948.

[2] R. A. Pucel, "Theory of the Esaki Diode Frequency Converter," *Solid State Electronics*, vol. 3, November 1961.

time-varying conductance that we wish to evaluate. The capacitance C represents the diode junction capacitance.

Dicke's method can only be applied if the diode presents a positive conductance to the LO. This condition can be fulfilled by loading the diode with a stabilizing conductance G_s which is slightly larger in magnitude than the most negative value of the diode conductance.[3] This conductance should be connected directly across the diode terminals, if possible. It also can be placed within a multiple of a half-wavelength from it (measured at the LO frequency) provided the diode does not oscillate. For computational purposes, it can be incorporated in the equivalent circuit as a shunt element across each of the three signal ports.

To evaluate the three mixer conductances, Dicke proposed that one measure the mixture admittance at the IF port using a bridge whose test frequency ω_2 is very small compared to the LO frequency. This insures that for all practical purposes the generated RF and image frequencies can be assumed equal to the LO frequency ω_0. For example, if the bridge frequency were 1 kc and the LO frequency were 500 Mc, the image, RF, and LO frequencies would be within 4×10^{-4} per cent of each other. Thus, if only the LO and the low-frequency bridge are applied to the stabilized diode, the resulting RF and image signals "see" the same circuit termination at their respective ports. Furthermore, this termination is just equal to the admittance presented to the diode by the LO source, since the RF, image, and LO frequencies are nearly equal. Let this admittance be denoted by y_s:

$$y_s = g_s + jb_s.$$

For convenience we include the susceptance of the diode capacitance at the RF-image-LO ports as part of b_s; that is, b_s contains the component $\omega_1 C\approx\omega_3 C\approx\omega_0 C$ in addition to any external susceptance presented by the LO circuit.

Under the above measurement conditions, the output admittance Y_{out} presented to the bridge at the IF port is given by

$$Y_{out} = g_0' - \frac{2g_1^2(g_0' + g_s - g_2)}{(g_0' + g_s)^2 - g_2^2 + b_s^2} + j\omega_2 C, \quad (1)$$

where $g_0'=g_0+G_s$. By measuring the conductive part of Y_{out} for three independent LO admittances g_s, b_s, one obtains three equations which can be solved for the unknowns g'_0 (hence g_0), g_1^2, and g_2. The junction capacitance C can be obtained from the imaginary part of Y_{out} for any of the three independent terminations. In the Dicke method the three independent RF-image port terminations are determined by a

[3] In practice, to insure stability, G_s may also have to satisfy other inequalities determined by the (neglected) series resistance and inductance. For further details see, for example:
U. S. Davidsohn, Y. C. Hwang, and G. B. Ober, "Designing with tunnel diodes," *Electronic Design*, vol. 8, pt. 2, pp. 66–71; February 17, 1960.
"Measurement of the equivalent circuit parameters of tunnel diodes," *General Radio Experimenter*, vol. 34, pp. 3–8, July–August, 1960.

simple standing-wave measurement in the LO circuit. To show how this is accomplished, we must first describe the test procedure in some detail.

The LO source driving the stabilized diode is connected to the latter through a matched transmission line or waveguide and a tuner as indicated in Fig. 2(a). The tuner

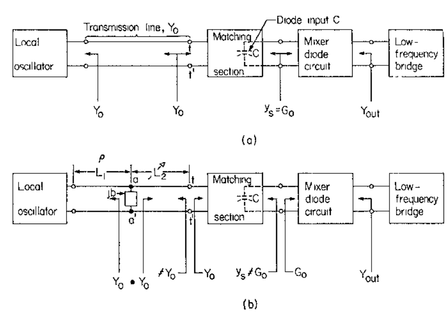

Fig. 2—The Dicke measurement circuit. (a) Matched conditions. (b) Mismatched conditions.

is assumed to be lossless, or nearly so. The diode input capacitance C is considered part of the tuner. The diode bias voltage is set to the desired operating point, and the LO output is adjusted to some prescribed amplitude as registered, for example, by the change in diode bias current. The tuner is then adjusted for a match in the LO line.[4] Since this adjustment may affect the LO excitation at the diode, it may be necessary to readjust the LO to restore the bias current to its previous value.

It can be shown[1,2] that the average (large-signal) conductance G_0 presented by the diode to the tuner is given by

$$G_0 = g_0' - g_2. \quad (2)$$

Since the tuner is lossless, under matched conditions the LO admittance presented to the diode at the tuner output terminals is real and obviously equal to G_0. That is,

$$y_s = g_s + jb_s = G_0 = g_0' - g_2. \quad (3)$$

Next an arbitrary susceptance jb, such as produced by a stub, is applied across the LO line section at, say, the plane a-a' indicated in Fig. 2(b). The LO output is readjusted, if necessary, to restore the previous bias current. This insures that the LO excitation at the diode terminals remains constant. Thus the admittance at the tuner input terminals t-t' and at the susceptance plane "looking" toward the diode is the same as in the matched condition, namely Y_0, the line admittance. In other words, the VSWR in the line section L_2 is still unity. However, this is not the case for the VSWR in the line section L_1 between the LO source and the susceptance plane. We shall denote this VSWR by ρ. Clearly, the admittance y_s "looking" back to the LO from the

[4] It is permissible in this case to speak of a match because the diode has been rendered passive by the stabilizing conductance.

diode terminals is no longer equal to G_0, the value established in the previous step.

Suppose now that with the susceptance in place the line section L_2 is varied in length, say, by a line stretcher. Since this section sees a match at its output terminals t-t' (the tuner is unaltered), it follows that the admittance presented to the LO does not change in this operation; that is, the LO excitation at the diode terminals is independent of L_2. On the other hand, the admittance looking back from the diode terminals does vary with L_2 and, in fact, traces the circle shown in Fig. 3, as is well known from transmission line theory.[5]

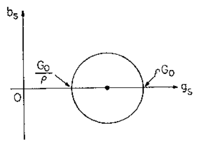

Fig. 3—Locus of the admittance y_s.

Using the equation for this circle one may eliminate the term b_s^2 in (1). The real part G_{out} of the resulting expression takes the form

$$G_{out} = g_0' - \frac{2g_1^2(g_0'+g_s-g_2)}{(g_0'^2-g_2^2-G_0^2)+g_s[2g_0'+G_0(\rho+1/\rho^{-1})]}. \quad (4)$$

This expression applies to any setting of the line stretcher. Since G_{out} is an increasing function of g_s, it follows that as the line stretcher is varied, the highest and lowest values of G_{out} measured by the bridge, which we denote by g_h and g_l, correspond to the highest and lowest values of g_s, namely, ρG_0 and G_0/ρ, respectively. It also is evident that G_{out} measured in the previous matched condition, which we denote by g_m, corresponds to $g_s = G_0$, $\rho = 1$.

Applying these three sets of conditions to (4) and using the identity for G_0 given by (2), we obtain three simultaneous equations which we can then solve for the mixer conductances. We obtain

$$g_0 = g_0' - G_s \quad \text{(a)}$$
$$g_1^2 = g_0'(g_0' - g_m) \quad \text{(b)}$$
$$g_2 = \frac{1-k}{1+k}\left(\frac{\rho+1}{\rho-1}\right)g_0', \quad \text{(c)} \qquad (5)$$

where

$$k = \frac{g_m - g_l}{g_h - g_m} \quad \text{and} \quad g_0' = g_h + \frac{g_h - g_l}{\rho k - 1}.$$

It is unnecessary to resolve the algebraic sign of g_1, since only $|g_1|$ or g_1^2 appear in equations pertaining to mixers.[2,6] Thus the desired mixer conductances can be calculated using four easily measured quantities.

Several precautions should be observed using Dicke's method. For example, an attenuator should be inserted between the LO and the standing-wave indicator to isolate the line from any admittance fluctuations of the LO. The attenuator can also be used to vary the oscillator output. It might be mentioned at this point that the tuner need not be connected directly to the diode terminals but can be any multiple of a half wavelength away from the diode reference plane. The audio bridge signal amplitude should be no greater than about 5 mv p-p to insure linear operation at the IF terminals.

A schematic diagram of a possible mixer test jig is shown in Fig. 4. The blocking capacitor C_b isolates the bias supply from the LO circuit. A path for direct current must exist in the bridge circuit, however.

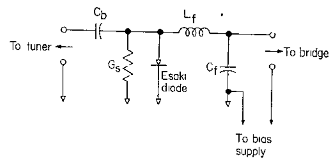

Fig. 4—A possible mixer test circuit for the Dicke method.

The choke L_f and capacitor C_f decouple the bridge from the LO circuit. The reactance of L_f and the susceptance of C_f should be large at the LO frequency but small at the bridge frequency. If C_f is too large, one must reduce the measured output capacitance by the value of C_f to obtain the true diode capacitance C.

In closing it should be mentioned that if the diode series impedance is not negligible or if the diode capacitance is voltage dependent, the more general but less convenient form of Dicke's method must be employed.[1]

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[5] W. C. Johnson, "Transmission Lines and Networks," McGraw-Hill Book Co., Inc., New York, N. Y.; 1950.

[6] C. S. Kim and J. E. Sparks, "Tunnel Diode Converters," *Proc. Natl. Electronics Conf.*, vol. 16, pp. 791–800, 1960.

A Versatile Phase Measurement Method for Transmission-Line Networks*

Current phase-measurement methods[1] for transmission-line circuits fall into two general categories. The first is the comparison method, where the path under measurement is compared with another calibrated, variable one. The phase shift through the variable path is adjusted to equal that of the unknown path, and this condition is shown by a null detector. Null indication by both phase and amplitude adjustment is usually required; however, null indication by phase adjustment alone[2] can be effected. The second category of phase measurements includes those measurements which provide voltage or meter output indication of phase angle. This category includes direct high frequency phase-detector circuits of limited frequency range or frequency-conversion methods where analog or digital circuits measure phase at a converted lower frequency.

The new method to be described here is an extension of the second category, which is the meter or voltage indication of phase. It is capable of direct measurement over a broad frequency range. Fig. 1 shows the arrangement of the phase meter equipment. The RF signal source for the phase measurement is amplitude modulated (sine wave or square wave). This modulation signal is carried as a sync to the synchronous detector. The RF signal is split, with one path going directly to one end of the slotted line as a reference phase signal and the other path going through the network under measurement. The output of the latter network feeds the other end of the slotted line as the unknown phase signal.

Fig. 2 develops the relations between the two RF field vectors, the individual square detector probe outputs, and the differential output of the two probes constituting the phase detector. The independent variable is the relative phase angle θ of the two RF signals for a fixed position X in the slotted line or the converse, variable position with fixed phase angle.

The vector diagram shows the resultant field vector E_t vs the RF phase angle θ. With a square law detector, the cosine law of triangles shows the output to be a constant term $(E_1^2+E_2^2)$ plus one varying with the cosine of phase angle θ. This relation is plotted for equal and unequal reference and unknown vectors E_1 and E_2.

The differential output $(V_a - V_b)$ of a pair of spaced probes is proportional to the sine of the phase angle. This allows the use of calibrated scales on the meter. The positive and negative output is provided by using a modulated RF test signal and a synchronous detector at the output of the amplifier. The meter and voltage output is calibrated by moving the phase detector successively to positions for the maximum positive and negative outputs. The amplifier gain and a balance control are adjusted so that these two outputs are fixed meter currents labelled $+90°$ and $-90°$. After calibration, the phase detector is positioned at the exact center of the slotted line to read the absolute phase difference of the two input signals referred to slotted line input terminals. The meter indication covers a 180° sector; to determine which of the two possible sectors the phase lies in, the phase detector is moved toward the unknown input. If meter motion is positive, the meter indication is correct. If a negative motion is obtained, the phase lies in the 90° to 270° sector and the actual angle is 180° minus the angle indicated on the meter.

High resolution is obtained by operating the phase meter near a zero and increasing the gain by definite amounts corresponding to meter scales; typical ranges are $\pm 90°$, $\pm 20°$, $\pm 6°$, $\pm 2°$, etc. In this mode of operation the distance of the phase detector from the center of the line is converted to a phase

* Received by the PGMTT, June 6, 1961.

[1] F. E. Terman and J. M. Pettit, "Electronic Measurements," McGraw-Hill Book Co. Inc., New York, N. Y., p. 267; 1952.

[2] S. D. Robertson, "A method of measuring phase at microwave frequencies," *Bell Sys. Tech. J.*, vol. 28, pp. 99–103, 1949.